Momentum distribution of electrons in the t-J model

1992 J. Phys.: Condens. Matter 4 L461

(http://iopscience.iop.org/0953-8984/4/35/002)

J. Phys.: Condens. Matter 4 (1992) L461–L466. Printed in the UK

LETTER TO THE EDITOR

Momentum distribution of electrons in the t–J model

Y Chen† and D K K Lee‡§

†Department of Mathematics, Imperial College, 180 Queen's Gate, London SW7 2BZ, UK

‡Department of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Received 29 June 1992

Abstract. We consider the electronic momentum distribution in systems displaying spin–charge separation. In particular, we consider the t–J model in the large-N limit. Assuming that there is no Bose condensation of holons at low temperatures in one and two dimensions, we find that the momentum distribution of the electron shows no Migdal discontinuity in one and two dimensions. In three dimensions, the Migdal discontinuity prevails as a signature of a Fermi-liquid state due to the presence of a holon condensate.

It was suggested by Anderson [1] that the electronic momentum distribution $n_{\boldsymbol{k}}$ in strongly correlated Fermi systems such as the hole-doped cuprates has no Migdal discontinuity [2] and should be described via the theory of the Luttinger liquid [3]. This conjecture has been demonstrated in one dimension where exact solutions have been found for the Hubbard model [4] and the supersymmetric point ($J = 2t$) of the t–J model [5]. The separation of the spin and charge degrees of freedom is a common feature in these models and is indeed generic to a whole class of Luttinger liquids. How is the momentum distribution affected by this phenomenon? In the absence of exact results for higher dimensions, it is interesting to calculate the momentum distribution in theoretical models where spin–charge separation is expected. Indeed, such a model has been studied in a previous paper by one of us [6]: in the large-N limit of the t–J model at $J = 2t$ in two dimensions, the spin and charge are not confined to form an electronic quasiparticle since the attraction between them decays with their separation as $1/r^4$. At this special large-N, $J = 2t$ limit, the bosonic holons experience no direct interactions and so they undergo Bose–Einstein condensation at sufficiently low temperatures with a free-particle spectrum—$\epsilon_{\boldsymbol{k}} = k^2/2m$. To discuss spin–charge separation, we write the electronic creation operator as

$$\bar{c}^\alpha = y^0 \bar{y}^\alpha$$

where y^0 annihilates a charged holon and $\bar{y}$ creates a neutral spinon with spin $\alpha = \uparrow$ or $\downarrow$. Bose condensation of the holons, i.e. $\langle y_0 \rangle = \sqrt{\rho_0} \neq 0$, implies that

$$\bar{c}^\alpha \to \sqrt{\rho_0} \mathrm{e}^{i\theta} \bar{y}^\alpha$$

§ Correspondence address.

(where θ is a phase field) so that the electronic momentum distribution would follow that of the spinon. In terms of the electronic spectral function, this implies a quasiparticle pole (in addition to an incoherent background due to real spinon–holon production). Thus a Migdal discontinuity is expected. Similarly, we expect this Fermi-liquid property to survive in high dimensions where there is a finite degeneracy temperature for the bosonic holons [7].

However, in one and two dimensions, the long-wavelength fluctuations of the phase θ destroy the holon condensate. In this letter, we use a large-N expansion to discuss the electronic momentum distribution in one and two dimensions for the particular cases when there is no Bose condensation—at zero temperature in one dimension and at finite temperatures in two dimensions below the Kosterlitz–Thouless transition [8] for the holons. We find that there is no Migdal discontinuity in the absence of Bose–Einstein condensation.

Consider first the t–J model away from the supersymmetric point in two dimensions. The imaginary-time Lagrangian reads

$$\mathcal{L} = \sum_{a=0,\uparrow,\downarrow} [\bar{y}^a \partial_\tau y^a + \tfrac{1}{2} J \overline{D_\mu y^a} D_\mu y^a] - (t - \tfrac{1}{2}J) \sum_{\alpha=\uparrow,\downarrow} [\bar{y}^\alpha y^0 \bar{y}^0 y^\alpha + \partial_\mu(\bar{y}^\alpha y^0)\partial_\mu(\bar{y}^0 y^\alpha)] \qquad (1)$$

with the constraint

$$\sum_{a=0,\uparrow,\downarrow} \bar{y}^a y^a = 1.$$

The gauge-covariant derivative D_μ is defined as

$$\begin{aligned} D_\mu y^a &= (\partial_\mu + \mathrm{i}A_\mu) y^a \\ A_\mu &= -\mathrm{i} \sum_{a=0,\uparrow,\downarrow} \bar{y}^a \partial_\mu y^a \end{aligned} \qquad (2)$$

reflecting the invariance of the t–J Hamiltonian with respect to phase transformations in the bosonic (y^0) and fermionic (y^α) slaves. The identification of A_μ is enforced by the field equation: $\partial\mathcal{L}/\partial A_\mu = 0$, similarly to in the treatment of the bosonic non-linear σ-model [9]. We may use the number constraint to eliminate the fermionic variables in the second term (which represents contact interaction) in the above Lagrangian. We then find an explicit repulsion term with strength $\propto (2t - J)$ between the bosonic slaves when $2t > J$. From the conventional theory of interacting Bose systems [10], we expect this to modify the behaviour of the low-lying bosonic excitations—from the free-particle spectrum at $2t = J$ to a collective branch $\Omega_k = c|k|$ where c can be identified as the sound velocity. Off-diagonal long-range order [11] and hence holon condensation is not expected to survive at finite temperatures in the presence of repulsive interactions. Instead, a power-law decay in the holon correlation function $\mathcal{G}^{\mathrm{B}}(x)$ is expected. (For $J > 2t$, the bosonic slaves attract each other and, indeed, there is phase separation of the holes from the spins at sufficiently large J/t [12].) We will now consider how this affects the electronic momentum distribution of the system in the large-N limit.

In our large-N extension of the model, we have N species of holons and spinons: $y^{0,\alpha} \to y_r^{0,\alpha}$ $(r = 1, \ldots, N)$. The number constraint on each site is now relaxed to

$$\sum_{r=1}^{N} \sum_{a=0,\uparrow,\downarrow} \bar{y}_r^a y_r^a = N.$$

As we are only considering a long-wavelength expansion, we neglect the derivative interaction in equation (1). The remaining term in the interaction is re-scaled and becomes

$$\frac{2t-J}{2N} \sum_{r,r'} \sum_{\alpha=\uparrow,\downarrow} (\bar{y}_r^\alpha y_r^0)(\bar{y}_{r'}^0 y_{r'}^\alpha). \tag{3}$$

At large N, the effect of the fluctuations in the gauge field is $O(1/N)$ [13] and so we will consider only its saddle-point configuration in the large-N limit. The leading contribution to the electronic Green's function is then the product of the spinon–antiholon Green's functions:

$$\mathcal{G}^{\mathrm{e}}_{\alpha\beta}(\boldsymbol{x}_{12}, \tau_{12}) = -\langle y^0(1)\bar{y}^\alpha(1) y^\beta(2)\bar{y}^0(2)\rangle \sim \mathcal{G}^{\mathrm{F}}(\boldsymbol{x}_{12}, \tau_{12})\mathcal{G}^{\mathrm{B}}(\boldsymbol{x}_{21}, \tau_{21})\delta_{\alpha\beta}. \tag{4}$$

We will consider the low-temperature regime where the spinons can be described as a degenerate Fermi gas and the holons possess power-law phase coherence in a Kosterlitz–Thouless phase: $0 < T \ll T_{\mathrm{KT}} \ll \epsilon_{\mathrm{F}}/k_{\mathrm{B}}$ where T_{KT} is the boson coherence temperature which we identify with the Kosterlitz–Thouless temperature and ϵ_{F} is the spinon Fermi energy. We will take

$$\mathcal{G}^{\mathrm{F}} = \left[\mathrm{i}\omega - \left(\frac{k^2}{2m_{\mathrm{F}}} - \epsilon_{\mathrm{F}}\right)\right]^{-1}$$

and neglect the thermal smearing of the spinon Fermi surface which only occurs over a small momentum range near the Fermi level for $k_{\mathrm{B}}T/\epsilon_{\mathrm{F}} \ll 1$. We will also adopt the Kosterlitz–Thouless model for our holons and use the conventional expression [14]: $\mathcal{G}^{\mathrm{B}}(\boldsymbol{x}, \tau) \sim \rho_0 \mathrm{e}^{-Q(\boldsymbol{x},\tau)}$ with

$$Q(\boldsymbol{x}, \tau) = \frac{m_{\mathrm{B}}c^2}{\rho_{\mathrm{s}}} \int_{|\boldsymbol{k}|<\Lambda} \frac{\mathrm{d}^2k}{(2\pi)^2} T \sum_{n=-\infty}^{\infty} \frac{1-\cos(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_n\tau)}{\omega_n^2 + c^2k^2} \tag{5}$$

where ρ_0 is the number density of the condensate (proportional to the fraction of holes δ),

$$c \simeq [(2t-J)\rho_0/m_{\mathrm{B}}]^{1/2}$$

is the sound velocity and ρ_{s} is the superfluid density defined as the component that does not carry any momentum in motion. (Λ is an ultraviolet cut-off and $\omega_n = 2\pi nT$ are the Matsubara frequencies.) This assumes a phonon-like spectrum which is generic to repulsive Bose systems at energies below an interaction scale $\sim (t - \frac{1}{2}J)\rho_0$. (At shorter wavelengths and higher energies, we expect to see free-particle behaviour.)

The momentum distribution for the physical electrons is given by

$$n_{\boldsymbol{k}} = \int \mathrm{d}^2 x \sum_{\alpha} \mathcal{G}^{\mathrm{e}}_{\alpha\alpha}(\boldsymbol{x}, -0)\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}. \tag{6}$$

In the temperature range considered,

$$\begin{aligned} \mathcal{G}^{\mathrm{F}}(\boldsymbol{x}, -0) &= \frac{k_{\mathrm{F}}^2}{2\pi} \frac{J_1(k_{\mathrm{F}} x)}{k_{\mathrm{F}} x} \\ \mathcal{G}^{\mathrm{B}}(\boldsymbol{x}, +0) &\sim \rho_0 \left(\frac{l}{x} \right)^{\eta(T)} \end{aligned} \tag{7}$$

where $l \propto (c/T)$, $x = |\boldsymbol{x}| \gg l$ and $\eta(T) = m_{\mathrm{B}} T / 2\pi\rho_0$ [14]. If we take the Kosterlitz–Thouless [8] model for our bosons, the transition to the totally disordered state occurs when η approaches $1/4$ from below. The momentum distribution is now given by

$$n_{\boldsymbol{k}} = \rho_0 (k_{\mathrm{F}} l)^{\eta} \int_0^{\infty} \frac{\mathrm{d}s}{s^{\eta}} J_1(s) J_0(ks/k_{\mathrm{F}}). \tag{8}$$

This integral is a representation of a hypergeometric function $F(a, b; c; z)$ in the variable $z = \kappa = |\boldsymbol{k}|/k_{\mathrm{F}}$. Note that the series representation of the hypergeometric function about $\kappa = 0$ has unit radius of convergence and we have to be careful in examining the behaviour of $n_{\boldsymbol{k}}$ near the Fermi surface. For $0 < \kappa < 1$ (and $-1 < \eta < 2$) [15]

$$n_{\boldsymbol{k}} = \rho_0 (k_{\mathrm{F}} l)^{\eta} \frac{\Gamma(1 - \eta/2)}{2^{\eta}\, \Gamma(1 + \eta/2)} F(1 - \eta/2, -\eta/2, 1, \kappa^2). \tag{9}$$

The behaviour of n_k as $\kappa \to 1^-$ can be mapped onto its series expansion around $\kappa = 0$ via the Kummer transformations [16]. Similarly, the distribution for $k > k_{\mathrm{F}}$ is given by

$$n_{\boldsymbol{k}} = \frac{\Gamma(1 - \eta/2)}{2^{\eta} \kappa^{2-\eta} \Gamma(\eta/2)} F\left(1 - \eta/2, 1 - \eta/2, 2, \frac{1}{\kappa^2}\right). \tag{10}$$

In this case, in order to determine the behaviour as $\kappa \to 1^+$, two consecutive Kummer transformations are needed to map the argument into the domain where the power series representation is valid. We find that the momentum distribution at the Fermi level is given by

$$n_{k_{\mathrm{F}}} = \rho_0 (k_{\mathrm{F}} l)^{\eta} \frac{\Gamma(\eta) \Gamma(1 - \eta/2)}{2^{\eta - 2} \eta^2 [\Gamma(\eta/2)]^3} \tag{11}$$

and this is approached symmetrically from above and below as a power law in $(1 - \kappa^2)$:

$$n_{\boldsymbol{k}} - n_{k_{\mathrm{F}}} \simeq \frac{\rho_0}{\pi} \left(\frac{k_{\mathrm{F}} l}{2} \right)^{\eta} \operatorname{sgn}(1 - \kappa^2) |\Gamma(-\eta)| \sin(\pi\eta/2) |1 - \kappa^2|^{\eta}. \tag{12}$$

The slope of n_k diverges at the Fermi level, resembling one-dimensional Luttinger systems. We expect to see this asymptotic behaviour for states whose energies are less than the interaction energy scale $(t - \frac{1}{2}J)\rho_0$ from the Fermi level. This is quite different from a distribution with a Migdal discontinuity which would be smooth over an energy scale of $k_B T$ only.

One can also calculate the specific heat of our model by integrating out the fermionic spinons and the Gaussian fluctuations in the holon phase. This gives an effective action for the $O(1/N)$ fluctuations of the gauge fields from which the free energy can be computed. The spinon contribution to the specific heat has linear temperature dependence while the holon contribution is proportional to T^2 in two dimensions [17].

Let us now consider the case in one dimension at $T = 0$. Holon condensation should again be absent. The asymptotic behaviour of the holon Green's function is [14]

$$\mathcal{G}_B(x, +0) \sim \rho |x|^{-\gamma} \tag{13}$$

where $\gamma = m_B c / 2\pi\rho$ for a system with total boson density ρ and sound velocity c. In this case [15],

$$n_k = \frac{2\rho}{\pi}(k_F \xi)^\gamma \int_0^\infty \frac{ds}{s^{1+\gamma}} \sin s \cos(|\kappa| s). \tag{14}$$

That is,

$$n_k = \rho (k_F \xi)^\gamma \frac{\sin(\gamma\pi/2)}{\pi\gamma} \Gamma(1-\gamma) \left[|1+\kappa|^\gamma \right. \\ \left. \times \operatorname{sgn}(1+\kappa) + |1-\kappa|^\gamma \operatorname{sgn}(1-\kappa) \right]. \tag{15}$$

Again there is no Migdal discontinuity in the absence of Bose condensation. A power law in the momentum distribution is also found in the exact solution of the $N = 1$ t–J model at the supersymmetric point [5]. In our heuristic large-N model, the holons behave as hard-core bosons at low densities and the sound velocity $c \sim \pi\rho/m_B$ so that $\gamma \to 1/2$ [18] (although we expect gauge fluctuations to modify this to approach the exact value of 1/8 at the supersymmetric point.)

In summary, we have calculated the momentum distribution for the physical electron in a large-N expansion. We find that the Migdal discontinuity is absent at low dimensions due to the infra-red divergence from the fluctuations of the holon phase. In contrast, holon condensation in three dimensions implies that the jump in the momentum distribution survives in agreement with our intuition.

One of us (DKKL) wishes to acknowledge the support of SERC grant GR/G02727.

References

[1] Anderson P W 1987 *Science* **235** 1196

[2] Anderson P W 1990 *Phys. Rev. Lett.* **64** 1839
Anderson P W and Ren Y 1990 *High Temperature Superconductivity—The Los Alamos Symposium* ed K Bedell, D Coffey, D Meltzer, D Pines and J R Schrieffer (Reading, MA: Addison–Wesley)

[3] Haldane F D M 1981 *J. Phys. C: Solid State Phys.* **14** 2585

[4] Frahm H and Korepin V E 1990 *Phys. Rev.* B **42** 10553
Kawakami N and Yang S-K 1990 *Phys. Lett.* **148A** 359
[5] Kawakami N and Yang S-K 1990 *Phys. Rev. Lett.* **65** 2309
[6] Chen Y, Föerster D and Larkin A *Phys. Rev.* B 1992 to be published
[7] Nagaosa N and Lee P A 1990 *Phys. Rev. Lett.* **64** 2450
[8] Berezinsky V L 1970 *Sov. Phys.-JETP* **59** 907; 1971 **61** 1144
Kosterlitz J M and Thouless D J 1973 *J. Phys. C: Solid State Phys.* **66** 1181
[9] D'Adda A, Lüscher M and di Vecchia P 1978 *Nucl. Phys.* B **146** 63
[10] Bogolyubov N N and Zubarev D N 1955 *Zh. Eksp. Teor. Fiz.* **28** 129 (Engl. Transl. 1955 *Sov. Phys.-JETP* **1** 83)
[11] Yang C N 1962 *Rev. Mod. Phys.* **24** 694
[12] Ogata M, Luchini M U, Sorella S and Assaad F F 1991 *Phys. Rev. Lett.* **66** 2388
[13] Chen Y and Foerster D 1992 *Phys. Rev.* B **45** 938
[14] Popov V N 1980 *Pis. Zh. Eksp. Teor. Fiz* **31** 560 (Engl. Transl. 1980 *Sov. Phys.-JETP Lett.* **31** 526)
Zabrodin A V and Ovchinnikov A A 1986 *Zh. Eksp. Teor. Fiz.* **90** 2260 (Engl. Transl. 1986 *Sov. Phys.-JETP* **63** 1326)
Popov V N 1987 *Functional Integrals and Collective Excitations* (Cambridge: Cambridge University Press)
[15] Gradshteyn I S and Ryzhik I M 1980 *Tables of Integrals, Series, and Products* (New York: Academic)
[16] Lebedev N N 1972 *Special Functions and Their Applications* (New York: Dover) ch 9
[17] Manning S and Chen Y 1992 in preparation
[18] Jimbo M, Miwa T, Môri Y and Sato M 1980 *Physica* D **1** 80